Electromagnetic Form Factors and the Hypercentral CQM

M. De Sanctis

INFN,Sezione di Roma1, Piazzale Aldo Moro, Roma (Italy)

M.M. Giannini, E. Santopinto and A.Vassallo

Universitá di Genova e INFN, Sezione di Genova,

via Dodecaneso 33, 16142 Genova (Italy)

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Abstract

New results about the electromagnetic form factors of the nucleon are obtained with a semirelativistic version of the hypercentral constituent quark model (hCQM) and a relativistic current. The complex structure of the constituent quarks is taken into account implicitly by means of phenomenological constituent quark form factors. We obtain a detailed reproduction of the experimental data up to $5\ GeV^2$, moreover our findings about constituent quark root mean square radii are of the same order than the recent ones obtained analyzing the proton structure functions.

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The new data on the ratio of the electric and magnetic form factors of the proton [1–3], showing an unexpected decrease with Q^2, have triggered again the interest in the description of the internal nucleon structure in terms of various effective models: bag models, chiral soliton models, constituent quark models, etc. .

Already in 1973 Iachello *et al.* [4] were able to obtain a good reproduction of all the existing nucleon form factor data using a Vector Meson Dominance (VMD) model introducing an intrinsic form factor to describe the internal structure of the nucleon. If one plots the ratio G_E/G_M, the results of the original fit decrease with Q^2 and cross zero at about $8\ GeV^2$. In 1996 Holzwarth [5] has shown that the simple Skyrme soliton model, with vector meson corrections and with the nucleon initial and final states boosted to the Breit Frame, leads to G_E^p that decreases with Q^2 and crosses zero at $10\ GeV^2$. In this case the crossing is due to a zero in the Skyrme model form factor as explained also in Ref. [6]. In 1995 using a constituent quark model Cardarelli *et al.* [7] have calculated the e.m. form factors of the nucleon in a light front approach fitting the SLAC data [8, 9] by means of form factors for the constituent quarks. Frank *et al.* [10] in 1996 have constructed a relativistic light cone constituent quark model and calculated the electric and magnetic form factors of the proton. If one plots their calculations as a ratio of the electric and magnetic form factors one can see a strong decrease with Q^2 due to the presence of a zero in the electric form factor at $Q^2 = 6\ GeV^2$ [11]. In 1999 [12] with a simple non relativistic quark model, the hCQM [13, 14], boosting the initial and final state to the Breit Frame and considering relativistic corrections to the non relativistic current [15] we have shown explicitly that the decrease is a relativistic effect [12, 16] and it disappears without these corrections [12, 16, 17]. This calculation makes use of the nucleon form factors previously determined in [15]. Using a chiral CQM and a point form dynamics the Pavia-Graz group [18, 19] has shown a good reproduction of the form factors and a decrease of the ratio up to $4\ GeV^2$. A similar reproduction [20] is obtained also within a Bethe Salpeter approach to a constituent quark model with an instanton based interaction [21]. Using the MIT Bag model a sharp decrease with Q^2 of the ratio is expected and a change of sign at $Q^2 = 1.5\ GeV^2$. The inclusion of the pion cloud not only improves the static properties of the model and restores the chiral symmetry, but also the behavior of the ratio G_E^p/G_M^p [22–24]. Interesting results are also obtained from LQCD calculations extrapolated to the chiral limit [25]. Finally we can say that the VMD fit by Iachello [26, 27] the extended VMD model by Lomon [28], the soliton model calculation by Holzwarth [5, 6], the calculation by Miller [11] and the relativistic two quark spectator model calculation by Ma *et al.*[29] describe the new Jlab data

quite well. Moreover the electric form factor of the proton is well described also by an explicit quark-diquark model [30]. For reviews on the subject the readers are referred to [31, 32].

In the following we present new results obtained with the hCQM, using a semirelativistic Hamiltonian and a relativistic quark current. Preliminary results have been presented at various Conferences [33–35].

First we briefly review the hCQM [13, 14]. The experimental 4 and 3 star non strange resonances can be arranged in $SU_{sf}(6)$ multiplets. This means that the quark dynamics has a dominant $SU_{sf}(6)-$ invariant part, which accounts for the average multiplet energies. In the hCQM it is assumed to be [13]

$$V(x) = -\frac{\tau}{x} + \alpha x, \tag{1}$$

where x is the hyperradius

$$x = \sqrt{\boldsymbol{\rho}^2 + \boldsymbol{\lambda}^2} \ , \tag{2}$$

with $\boldsymbol{\rho}$ and $\boldsymbol{\lambda}$ being the Jacobi coordinates describing the internal quark motion.
Interactions of the type linear plus Coulomb-like have been used since long time for the meson sector, e.g. the Cornell potential. Moreover this form has been supported by recent Lattice QCD calculations [36].
In the case of baryons a so called hypercentral approximation has been introduced [37, 38]; this approximation amounts to average any two-body potential for the three quark system over the hyperangle $t = arctg(\frac{\rho}{\lambda})$ and the angles Ω_ρ and Ω_λ, and it works quite well especially for the lower part of the spectrum [39]. In this respect, the hypercentral potential of Eq.(1) can also be considered as the hypercentral approximation of the two-body linear plus Coulomb-like potential. The splittings within the multiplets are produced by a perturbative term breaking the $SU_{sf}(6)$ symmetry, which, as a first approximation, can be assumed to be the standard hyperfine interaction H_{hyp} [40]. We consider the non strange baryons as bound states of three constituent quarks; in the baryon rest frame, the three quark Hamiltonian can be written as:

$$H_{nr} = 3m + \frac{\boldsymbol{p}_\rho^2 + \boldsymbol{p}_\lambda^2}{2m} - \frac{\tau}{x} + \alpha x + H_{hyp}, \tag{3}$$

where m is the quark mass (taken equal to $1/3$ of the nucleon mass) and $\boldsymbol{p}_\rho$, $\boldsymbol{p}_\lambda$ are the conjugate momenta of the Jacobi coordinates. The strength of the hyperfine interaction is determined in order to reproduce the $\Delta - N$ mass difference, the remaining two free parameters are fitted to the spectrum leading to the values $\alpha = 1.61 \ fm^{-2}$ and $\tau = 4.59$.

Keeping these parameters fixed, this non relativistic constituent quark model has been used to calculate various physical quantities of interest: the photocouplings [41], the electromagnetic transition amplitudes [42] and, introducing relativistic corrections to the one-body non relativistic current, also the elastic nucleon form factors [15] and the ratio between the electric and magnetic form factors of the proton [16]. We have shown that kinematical relativistic corrections (like boosts and a relativistic one body current with an expansion in the quark momenta up to the first order, keeping the exact dependence on the momentum transfer Q^2) are very important for the elastic form factors [15, 16] but give only minor corrections in the transition ones [43].

As a further step towards a full relativistic description, we have developed a semirelativistic version of the hypercentral constituent quark model. The resulting wave functions, boosted to the Breit frame, have been used to calculate the matrix elements of the relativistic one body current without any expansion in the quark momenta.

The potential of Eq. (1) has been fitted to all the 3 and 4 stars resonances using the correct relativistic kinetic energy [44]

$$H = \sum_{i=1}^{3} \sqrt{p_i^2 + m^2} - \frac{\tau}{x} \; + \; \alpha x + H_{hyp}. \tag{4}$$

The eigenequation of the Hamiltonian (4) is solved in the nucleon rest frame, the resulting spectrum is not much different from the non relativistic one and the parameters of the potential are only slightly modified, confirming that kinematic relativistic corrections are not very important for the spectrum. A complete description of a variational solution of this equation, using a hyperspherical formalism will be published elsewhere [44]. A first semirelativistic version of our hCQM, but without SU(6) breaking and so without the possibility of reproducing the spectrum, was first introduced by Traini and Faccioli [45] and the wave functions obtained in this way have given realistic results for structure functions [45–48] and also for the generalized parton distributions [49–51].

The baryon current is chosen to be the sum of the three elementary quark currents and it corresponds to the impulse approximation [15, 16]:

$$j_\mu^{(q)} = \bar{u}(\boldsymbol{p}')\gamma_\mu u(\boldsymbol{p}) \tag{5}$$

and the nucleon state in the rest frame is assumed to be [15]:

$$\Psi_{\boldsymbol{P}=0}(\boldsymbol{p}_\rho, \boldsymbol{p}_\lambda) = u(\boldsymbol{p}_1)u(\boldsymbol{p}_2)u(\boldsymbol{p}_3)\varphi(\boldsymbol{p}_\rho, \boldsymbol{p}_\lambda) \tag{6}$$

where $\varphi(\boldsymbol{p}_\rho, \boldsymbol{p}_\lambda)$ with $\boldsymbol{p}_\rho = (\boldsymbol{p}_1 - \boldsymbol{p}_2)/\sqrt{2}$ and $\boldsymbol{p}_\lambda = (\boldsymbol{p}_1 + \boldsymbol{p}_2 - 2\boldsymbol{p}_3)/\sqrt{6}$, is the eigenfunction of the Hamiltonian of equation (4); $u(\boldsymbol{p}_i)$ is the Dirac spinor of the i-th quark and $\boldsymbol{p}_i$ is momentum

of the i-th quark in the nucleon rest frame. The boosts to the Breit frame have been applied to the initial and final wave functions. With respect to our previous papers [15, 16], both the boosts on the spatial variables and on the Dirac indices are performed without any approximation. Moreover another important improvement is given by the use of semirelativistic wave functions.

The resulting theoretical form factors of the nucleon, calculated without free parameters, and without any expansion in the quark momenta, can be seen in FIG. 1 and 2.

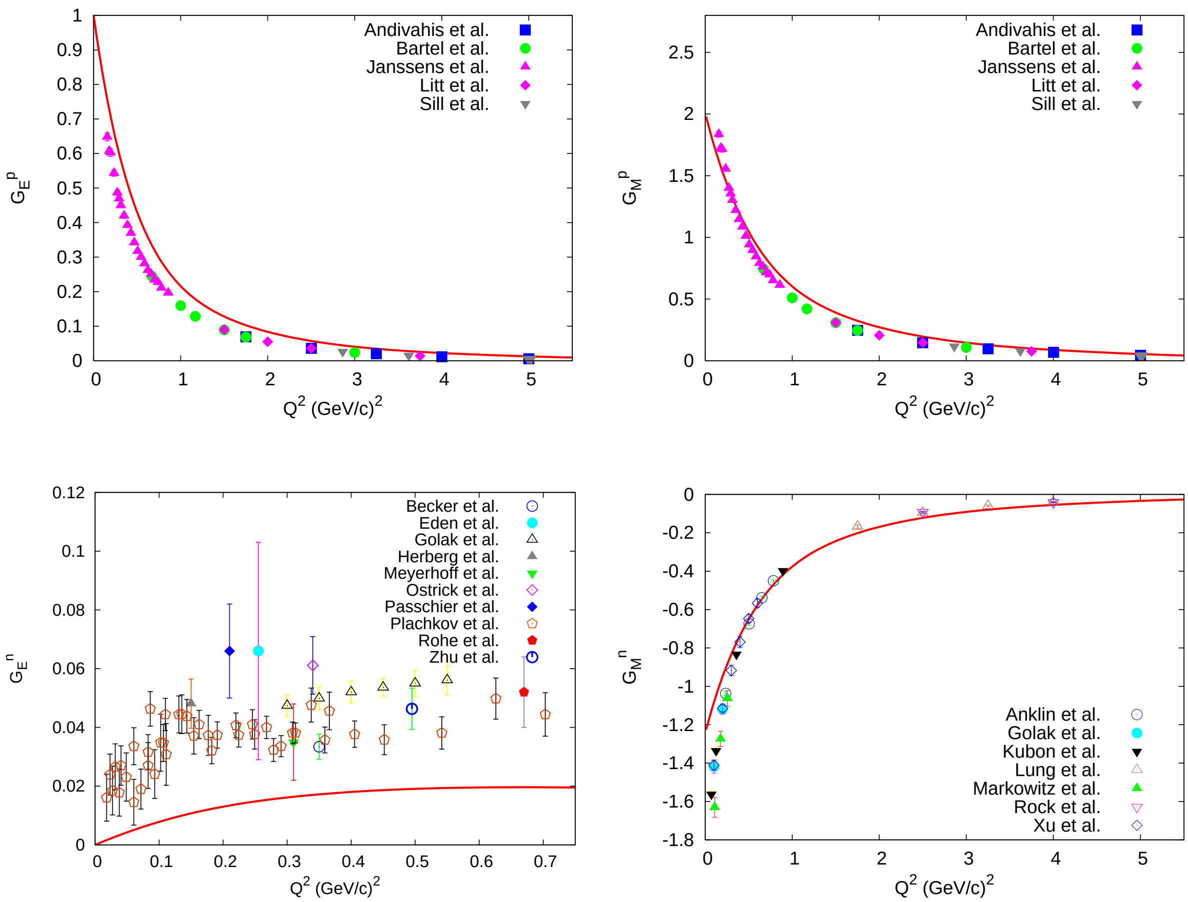

FIG. 1: (Color online) Elastic form factors of the nucleon. The solid line corresponds to the semirelativistic hCQM calculation. The experimental data for proton are taken from the reanalysis made by Brash *et al.* [52] of the data from [53–57]; for the neutron electric form factor, the experimental data are taken from [58–67], and for the neutron magnetic form factor, the experimental data are taken from [60, 68–73].

The results reported in Figure 1 show a quite good reproduction of the data even if some problems are still present especially at low Q^2. Nonetheless there is a great improvement in comparison with the non relativistic calculations of Refs [15, 16].

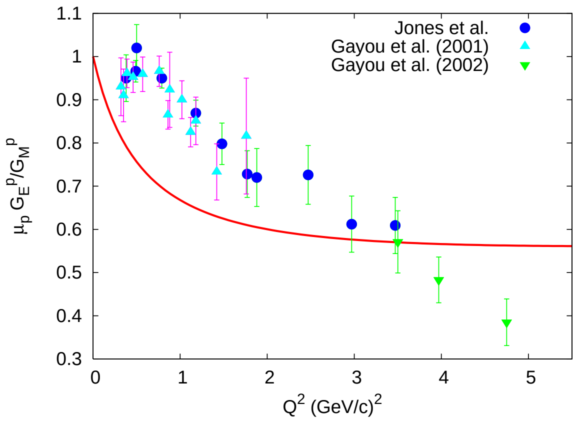

FIG. 2: (Color online) The measured ratio $\mu_p G_E^p/G_M^p$ compared with the semirelativistic hCQM calculation (solid line). The experimental data are taken from [1–3].

Since we consider constituent quarks, which can be in principle considered composite objects [74], we parametrize phenomenologically their structure by means of constituent quark form factors, as already done by other authors [7].

If the constituent quarks have electromagnetic form factors, the quark current can be written as:

$$j_\mu^{(q)} = \bar{u}(\boldsymbol{p}')\left[(F_1^q(Q^2) + \kappa_q F_2^q(Q^2))\gamma_\mu - \frac{1}{2m}\kappa_q(p+p')_\mu F_2^q(Q^2)\right]u(\boldsymbol{p}) \tag{7}$$

where F_1^q and F_2^q are the Dirac and Pauli form factors and κ_q is the anomalous magnetic moment of the q $(= u, d)$ quark. The constituent quark form factors are chosen as a linear combination of monopole and dipole Q^2 behavior. Fitting the free parameters to the reproduction of G_M^p, G_M^n, G_E^n and $\mu_p G_E^p/G_M^p$ we obtain the curves shown in FIG. 3 and 4

The resulting values of the constituent quarks mean square radii are:

$$< r^2 >_u = 0.10 \text{ fm}^2 \qquad\qquad < r^2 >_d = -0.09 \text{ fm}^2 \tag{8}$$

in good agreement with the findings of Petronzio *et al.* [74], who have been able to identify complex objects inside the proton. As it can be seen in FIG 3 and 4 the experimental data are very well reproduced. The goodness of the reproduction is emphasized by plotting the form factors divided by the dipole form.

With respect to the non relativistic case, the semirelativistic wave functions have more high momentum components. This fact, together with the application of exact boosts to the Breit Frame, leads to an improvement in the reproduction of the existing data on the electromagnetic form

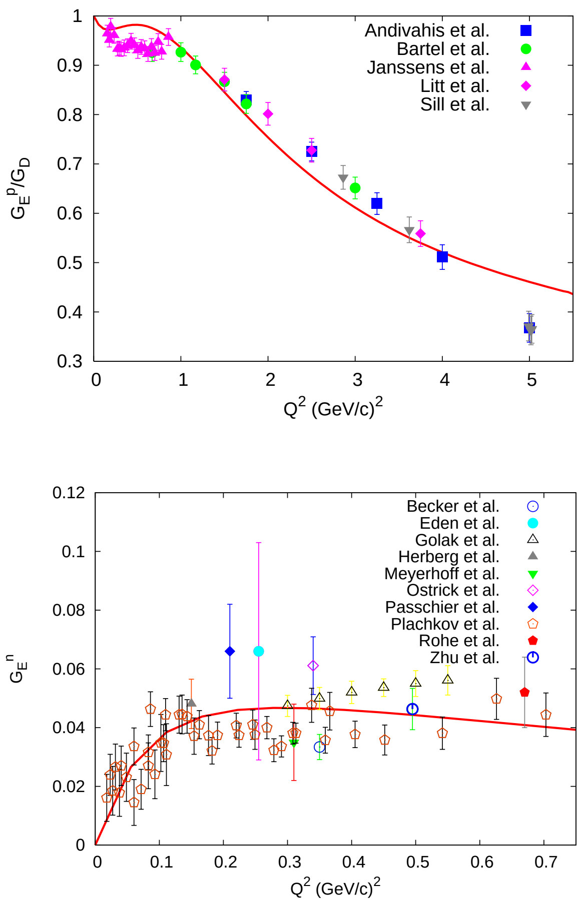

FIG. 3: (Color online) Elastic form factors of the nucleon. The solid line corresponds to the semirelativistic hCQM calculation with constituent quark form factors. The experimental data for proton are taken from the reanalysis made by Brash *et al.* [52] of the data from [53–57]; for the neutron electric form factor, the experimental data are taken from [58–67], and for the neutron magnetic form factor, the experimental data are taken from [60, 68–73].

factors. However a good description of the data is obtained only if phenomenological constituent quark form factors are introduced in the electromagnetic current. In this way we have a very nice agreement with the available experimental data up to $5\ GeV^2$. Moreover the dimensions of these composite constituent quarks are found in agreement with the recent analysis of the nucleon structure functions performed by Petronzio *et al*.

Finally, it results that for a good reproduction of the elastic form factor data both the relativistic effects and the composite nature of the constituent quarks have to be taken into account. We observe that such constituent quark form factors actually parametrize not only their structure but

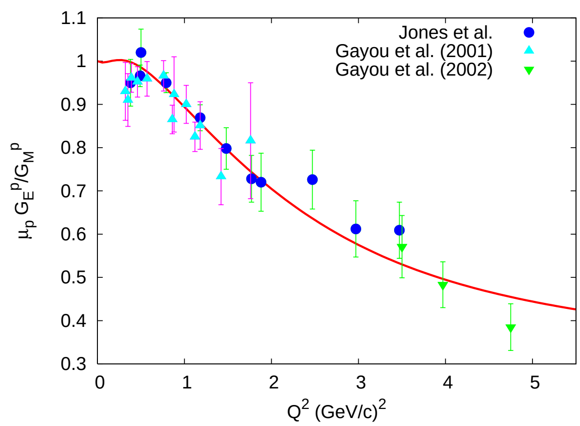

FIG. 4: (Color online) The measured ratio $\mu_p G_E^p / G_M^p$ compared with the semirelativistic hCQM calculation (solid line). The experimental data are taken from [1–3].

also all the relativistic effects which have not yet been included in our calculations.

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